1. Let $\gamma_r(t) = re^{it}$ be the circle of radius r. Describe $\int_{\gamma_r} \frac{1}{\sin(z)} dz$ as a function of r. (Take the domain of this function to be positive real numbers for which $\sin(r) \neq 0$. Give an exact formula if you can, otherwise give any description you can of what this function is like.)

2. Let $U = \{x + iy \in \mathbb{C} \mid -\pi/2 < x < \pi/2 \text{ and } -\cos(x) < y < \cos(x)\}$. Draw a picture of U. Let $V \subset U$ be a disk of radius 4. Can a holomorphic function $f : U \to \mathbb{C}$ have $f(U) = V$? Can a holomorphic function $f : \mathbb{C} \to \mathbb{C}$ have $f(U) = V$? Give reasons.

3. Fix a complex number a. Let $f : \mathbb{C} \to \mathbb{C}$ be the function defined by $f(z) = z^3 + az + 1$. Determine the largest open subset of $\mathbb{C}$ on which f is conformal.

4. Suppose that $g : \mathbb{C} \to \mathbb{C}$ is holomorphic with Taylor series $g(z) = a_0 + a_1 z + a_2 z^2 + \cdots$. Suppose furthermore that $|g(z)| \leq 1$ whenever $|z| \leq 1$. Show that $|a_k| \leq 1$ for all k.

5. Determine all biholomorphisms (i.e. holomorphic automorphisms) $f : \mathbb{C} \cup \{\infty\} \to \mathbb{C} \cup \{\infty\}$ that have $f(0) = 0$ and $f(\infty) = \infty$. Here $\mathbb{C} \cup \{\infty\}$ denotes the Riemann sphere, i.e. the extended complex plane.