Algebra Qualifying Exam
Fall 2015
3 hours

1. Classify groups of order 55 up to isomorphism. Give a presentation for each of the groups in your classification.

2. Let $R = \mathbb{C}[X, Y]$ and consider the ideal $I = (X, Y)$ as an $R$-module.

(a) Construct an exact sequence of $R$-modules

$$0 \to R \to R \quad R \to I \to 0.$$

(b) Prove that the sequence you constructed is not split.

3. Consider the ideal

$$I = (X^2 \quad Y, Y^2 \quad X) \quad \mathbb{C}[X, Y].$$

Find all maximal ideals of the quotient $\mathbb{C}[X, Y]/I$. (Find means give a set of generators.)

4. How many Sylow $p$-subgroups are there in $GL_2(\mathbb{F}_p)$?

5. Suppose $K$ is an extension of $\mathbb{Q}$ of degree $n$, and let $_1, \ldots, _n : K \to \mathbb{C}$ be the distinct embeddings of $K$ into $\mathbb{C}$. Let $\in K$