Algebra Qualifying Exam
August, 2012

Please answer all 10 problems and show your work. Each problem is worth 20 points. In your proofs, you may use any theorem from the syllabus for Algebra, except of course you may not use the fact you are trying to prove, or a mere variant of it. State clearly what theorems you use. Good luck.

1. Let G be a group of order pqr, where $p < q < r$ are distinct primes. Prove that a Sylow r-subgroup GL

5. Let F be a finite field of cardinality q and let V be a four-dimensional vector space over F. The group $V \simeq {}_4 F$ acts on V. Let U be a two-dimensional subspace of V. Compute the order of the subgroup $\{g \in GL\ V \ gU \ U\}$ and determine the number of two-dimensional subspaces of V VW=t3J SR

the ring of invariants.

(a) Prove that S is integral over R.

(b) Let k be a field and suppose that S is a finitely generated k-algebra, and that G acts on S vi k-algebra automorphisms. Prove that S is a finitely generated R-module.