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INTRODUCTION

Ambiguity aversion is one of the most investigated phenomenon in decision theory. Ambiguity refers to situations where a decision maker does not know the exact probabilities of some events. The claim that decision makers systematically prefer betting on events with known instead of with unknown probabilities, a phenomenon known as ambiguity aversion, was first suggested in a series of examples by Ellsberg (1961) and was soon proved to hold true in many experiments. The importance of Ellsberg's findings stems from the fact that they cannot be reconciled with individuals holding *any* subjective probabilities over events. Mainly motivated by Ellsberg's examples, several formal models have been proposed to accommodate ambiguity aversion. One of the most important models in the literature, known as Choquet expected utility (Schmeidler, 1989), assumes that decision makers hold nonadditive beliefs (called capacities), which overweight events associated with bad outcomes.

Ellsberg's experiments involve binary bets (that is, the ambiguous prospects have only two possible outcomes). Machina (2009) claims that there are some aspects of ambiguity aversion that arise only in the presence of nonbinary bets. For example, if there are three possible monetary outcomes $a > b > c$, then a decision maker may prefer ambiguity about the probabilities of receiving a and b to ambiguity about the probabilities of receiving b and c. Accordingly, Machina (2009) suggests some examples that involve three or more outcomes and shows that plausible attitudes toward ambiguity in these problems cannot be accommodated by Choquet expected utility. Baillon et al. (2011) show that Machina's examples pose difficulties not only for Choquet expected utility but for several other known models as well.[2,3] In a follow-up paper, Machina (2014) offers more thought experiments of nonbinary bets and explains why they pose new difficulties for Choquet expected utility as well as to some other models.

Machina's examples are in line with a well-established tradition of "puzzles" in decision theory: A theory implies a specific relationship between two choice problems, even though

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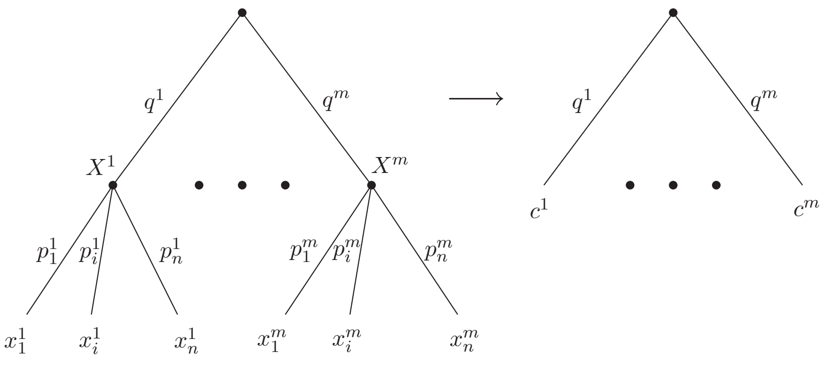

FIGURE 1

RECURSIVE EVALUATION OF A TWO-STAGE LOTTERY

thought or actual experiments systematically violate this relationship. Such are, for instance, the aforementioned Ellsberg's examples that challenge the subjective expected utility model of Savage (1954) and, in the context of decision making under risk, Allais (1953) paradox. In a similar way, Machina's examples challenge the links between different decision situations implied by Choquet expected utility.

In this article, we show that all of Machina's examples can be handled by the two-stage recursive ambiguity model of Segal (1987) and, moreover, that this can be done using the same functional form for all examples. According to the recursive model, ambiguity corresponds to the case where there is some set of states of the world and the decision maker does not know the exact probability distribution over these states. Instead, he has in mind a set of conceivable distributions and, furthermore, he is able to assign (subjective) probabilities to the different distributions in this set. For each distribution, the decision maker computes its certainty equivalent using some nonexpected utility functional. He then views the uncertain prospect as a lottery over these certainty equivalents and evaluates it using the same nonexpected utility functional. We provide some simple examples demonstrating that the recursive model is rich enough not to impose the links between different decision situations that exist in Choquet expected utility. While without further restrictions the recursive model is very general, we show that a single functional form can address all the aspects described in Machina's examples.

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TABLE 1
THE 50:51 EXAMPLE

Act	50 Balls		51 Balls	
	E_1	E_2	E_3	E_4
f_1	8,000	8,000	4,000	4,000
f_2	8,000	4,000	8,000	4,000
f_3	12,000	8,000	4,000	0
f_4	12,000	4,000	8,000	0

TABLE 2
THE REFLECTION EXAMPLE

	50 Balls		50 Balls	
Act	E_1	E_2	E_3	E_4
f_5	4,000	8,000	4,000	0
f_6	4,000	4,000	8,000	0
f_7	0	8,000	4,000	4,000
f_8	0	4,000	8,000	4,000

TABLE 3
THE SLIGHTLY BENT COIN PROBLEM

I	Black	White	*II*	Black	White
Heads	8,000	0	Heads	0	0
Tails	−8,000	0	Tails	−8,000	8,000

We obtain that f_1 □ f_2 but f_4 □ f_3.

3.2. *The Reflection Example.* Consider the acts shown in Table 2.

The two acts f_5 and f_8 reflect each other, and the decision maker should therefore be indifferent between them. Likewise, f_6 should be indifferent to f_7. As by the Choquet expected utility model f_5 □ f_6 iff f_7 □ f_8, it follows that f_5 □ f_6 (and f_7 □ f_8). Yet, as is argued by Machina (2009, section III), ambiguity attitudes may well suggest strict preference within each pair.

Let $\alpha, \beta, \gamma, \delta$ be a list of possible numbers of balls of the four types in the urn, where $\alpha + \beta = \gamma + \delta = 50$. Denote by $q(\alpha, \beta, \gamma, \delta)$ the probability the decision maker attaches to the event "the composition of the urn is $\alpha, \beta, \gamma, \delta$." We say that such beliefs are symmetric if

$$q(\alpha, \beta, \gamma, \delta) = q(\beta, \alpha, \delta, \gamma) = q(\gamma, \delta, \alpha, \beta) = q(\delta, \gamma, \beta, \alpha).$$

If beliefs are symmetric, then the recursive model implies f_5 □ f_8 and f_6 □ f_7, yet it does not require f_5 □ f_6. In fact, it can be shown that such indifference will *not* hold in general. For example, if $q(10, 40, 25, 25) = \frac{1}{4}$ then we have f_6 □ f_5.

3.3. *The Slightly Bent Coin Problem.* A coin is flipped and a ball is drawn out of an urn. You know that the coin is slightly bent (but you do not know which side is more likely or the respective probabilities) and that the urn contains two balls, each is either white or black. Which of the bets given in Table 3 do you prefer?

According to Machina (2014, section IV), it is plausible that an ambiguity averse decision maker will prefer Bets *I* to

TABLE 4
POSSIBLE PROBABILITY DISTRIBUTIONS

Case #	Pr(*head*), # of *black*	Prob.	hb	hw	tb	tw
1	$\frac{1}{2}+\varepsilon, \#b=2$	$\frac{q}{2}$	$\frac{1}{2}+\varepsilon$	0	$\frac{1}{2}-\varepsilon$	0
2	$\frac{1}{2}-\varepsilon, \#b=2$	$\frac{q}{2}$	$\frac{1}{2}-\varepsilon$	0	$\frac{1}{2}+\varepsilon$	0
3	$\frac{1}{2}+\varepsilon, \#b=1$	$\frac{1}{2}-q$	$\frac{1}{4}+\frac{\varepsilon}{2}$	$\frac{1}{4}+\frac{\varepsilon}{2}$	$\frac{1}{4}-\frac{\varepsilon}{2}$	$\frac{1}{4}-\frac{\varepsilon}{2}$
4	$\frac{1}{2}-\varepsilon, \#b=1$					

popular models, including Choquet expected utility. As argued by Machina, the reason is that these models impose too much separability in the way outcomes paid on different events are aggregated in the evaluation procedure. In this article, we show that all these issues can be accommodated by the two-stage recursive ambiguity model of Segal (1987) and, moreover, that this can be done using the same functional form for all examples. In other words, the recursive