

## TOPOLOGY QUALIFYING EXAM SYLLABUS, 2013

### **Algebraic Topology** (text: **Algebraic Topology** by Allen Hatcher)

- Basic point-set topology
- Classification of closed surfaces
- Fundamental group
- Covering spaces
- Homology (including simplicial homology, singular homology and cellular homology).
- Cohomology (including cohomology groups, the universal coefficient theorem, cup product, and cohomology ring).
- Duality theorems (including Poincaré duality, Lefschetz duality and Alexander duality).

### **Smooth Manifolds** (text: **Introduction to Smooth Manifolds** by J. Lee).

- Topological manifolds, smooth manifolds
- Smooth maps, covering maps, partitions of unity
- Tangent vectors (various definitions), pushforwards
- Computation in coordinates
- Tangent Bundle, vector fields, Lie bracket
- Vector Bundles, sections of vector bundles, bundle maps
- Cotangent Bundle, covectors, differential of a function, pullbacks
- Line integrals, conservative covector fields
- Inverse Function Theorem, Implicit Function Theorem, Rank Theorem
- Submersions, immersions, embeddings
- Level Set Theorems, embedded submanifolds, immersed submanifolds
- Whitney Embedding Theorems, Whitney Approximation Theorem
- Tensors, tensor fields, symmetric tensors, Riemannian metrics
- Differential forms, alternating tensors, wedge product, exterior derivatives
- Orientation, boundary orientation, Riemannian volume form
- Integration on manifolds, Stokes Theorem, integration on Riemannian manifolds
- DeRham cohomology, deRham's Theorem
- Lie groups, Lie algebras, invariant vector fields
- Group actions, Quotient Manifold Theorem, homogeneous spaces
- Symplectic form
- Morse Theory