# Algebra Qualifying Examination, June 2019

**Instuctions:** This is a 3 hour examination. In the problems below, all rings are commutative with identity. This is a closed book exam, also no notes, searching the web, or otherwise consulting external sources. Good luck!

1. Let $G$ be a group of order 108. Show that $G$ has a normal subgroup of order 9 or 27.
2. Let $R$ be a ring, and let $D$ be the set of all $x$ 2 $R$ such that $x$ is a zero divisor or $x = 0$. Show that $D$ is a union of prime ideals. (Hint: consider the set of all ideals contained in $D$. Show that contains maximal elements and every maximal element of is prime.)